COMPUTER ASSISTED MODELS USED IN THE SOLUTION OF WAREHOUSE LOCATION-ALLOCATION PROBLEMS

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ABSTRACT

This paper presents a general overview of the various types of computer based algorithms solving the Warehouse Location-Allocation problem. Available heuristic algorithms are compared with exact optimization models for accuracy of the solution and time to make the computer runs. Salient features and applicability of the various models are presented. A review of the available software to solve this type of problem is given. Finally, the future trends in developing models to solve these problems is discussed.

INTRODUCTION

Changes in plant capacity and the economics of warehousing and transportation, shifting markets and competitors' action all impact on the effectiveness of a physical distribution system. Firms have found that a comprehensive computerized model of a distribution system can be an effective tool for dealing with these complexities, furthermore, that it is able to escape conventional analysis based on traditional tools.

The term "distribution planning" is defined, then costs which can be identified for a warehousing system are delineated and discussed. Restrictions experienced in this problem are noted and significant questions given. There is a review of techniques which can be used to solve the problems of distribution planning.

THE DISTRIBUTION PLANNING MODEL

The Problem of Distribution Planning

"Distribution planning" refers to intermediate-term (one to five years) planning of the configuration and flow aspects of distribution systems, involving modifications to an existing distribution system or the design of a new one. Several questions must be answered before designing the system: (1) How many warehouses should there be, (2) Where would they be located, (3) What size should each each warehouse be, (4) Which warehouse should service which customer, (5) How should the output of each plant be allocated relative to warehouses and customers, and (6) What would be the effect upon cost and customer services, of a new system in some future target period? These answers must be determined simultaneously, rather than in isolation.

Cost Elements of the Distribution Planning Model

The distribution planning model has four independent sources of cost: (1) All transportation costs, (2) All warehouse and inventory costs, (3) Costs and savings of expanding, opening, and closing warehouses, and (4) Production costs at the plants.

Model Constraints

The objective of distribution planning is to minimize the sum of these costs, subject to: (1) Stipulated plant production capacities must not be exceeded, (2) Each warehouse size must be between prescribed limits, (3) Sometimes, selected customers must be served by a single warehouse for certain products, (4) A warehouse is eligible to serve a customer only if it is sufficiently close so that the transit times, under economical delivery modes, are in accord with the desired level of customer service, (5) All forecast demands must be satisfied, and (6) Any other desired constraints such as lower and upper limits on the number of open warehouses.

SALIENT FEATURES TO BE MODELED

A distribution planning model statements and assumptions about a system, describing it in detail and producing a formalized representation for answering the questions above. Designing the computer model is a compromise between detail and economy of computer usage. Essential features of the system must be reflected in sufficient detail so that the model will yield valid conclusions, acceptable to management. The level of detail should not make the model's data requirements so large that available computational technique can not solve it at reasonable expense. Common types of problems encountered when constructing these models are:

Multiple Products

Different sizes and packages are best viewed as different products. When they share the use of production and/or distribution facilities, it is necessary to treat multiple products in the model.

Stages of Distribution

The stages of a distribution system are: (1) the Plant/Warehouse, (2) the Warehouse/Customer stage. To model the system, these stages should be modelled separately. Early distribution system models treated only a single stage distribution. For the plant-to-warehouse stage, the customers must be collapsed into warehouses, allocating demands to the warehouse a priority. If the warehouse-to-customer stage is selected for study, plants must be collapsed into warehouses by guessing the optimal allocation to the warehouses or by treating plant capacities as infinite.

Capacities for Plant and Size Limits for Warehouses

A distribution system model should control the size of a warehouse, expressed in terms of annual thoroughput volume (eg. cases). Minimum and maximum size limits can be used to keep the thoroughput of an existing facility within the range over which no major change is needed in floor space or material handling methods.

Warehouse Economies of Scale and Fixed Charges

Decisions to open or close warehouses result in fixed charges which cannot be apportioned to direct sources and are difficult to compute. Similarly, economies of scale are often significant enough to necessitate their inclusion in the model.

Each Customer Serviced by a Single Warehouse.

Many firms service each customer from a single distribution center for as many products as in feasible. To reflect this policy, a model must include the multiple choice aspect of assigning each customer to a single distribution center.

Shipments to Some Customers must Reflect Brand or Source Loyalty

On occasions, it is desirable for specific plants to be identified by customers as the source. The freight moves from plant to customer under a "storage in transit" privilege. This leads to lower freight rates than the sum of plant-to-warehouse and warehouse-to-customer rates. However the plant origin of transit to the customer must be tracked with care.

CRITERIA FOR AN EFFICIENT COMPUTATIONAL METHOD

Realistic models are useless unless a computational method is available to solve it with efficiency and produce insights and conclusions. A computational method should meet the following criteria:

(1) It should truly optimize.
(2) Computer costs should be moderate
(3) It should facilitate multiple secondary optimization runs, such as:

i. Sensitivity analysis runs, errors or changes in the data are explored to determine how "robust" the model implications are.
ii. Trade off analysis runs develop the trade-offs between total cost and quality of customer service; capital expenditure and operating cost reductions; energy consumption and total cost.
iii. Capacity valuation runs, assess marginal value of increased capacity at plants and warehouses.
iv. Implementation priority analysis runs identify specific changes in the distribution system that account for the largest potential savings.

EXISTING COMPUTATIONAL METHODS

Classification as Macro or Micro Models

Macro models link various functions of the organization such as marketing, production capacity, and planning to distribution. The models sacrifice detail, since it is impossible to maintain all the details of each function. Macro models exist to gain insights into the functioning of an organization and even a whole industry. These models throw light on the assumptions and reveal the limitations of the usable models. Macro models are wider in scope and involve long term decisions. Plant location is typical of such a decision involving intangible factors and uncertainties.

Micro models involve short term decisions and look into quantifiable variables. Vehicle routing is one such model. A clear dividing line between macro and micro models does not exist; it is more of a continium. Somewhere in the continium lies the depot location problem. Depot location is related to other functions of the organization and involves complications such as qualitative variables. Yet topography and distance play such a prominent role that the problem as viewed by many researchers, lend itself to modelling.

Depot location is a medium term problem of deciding how many depots to operate, and how to serve the customers from them. The major approaches to the problem are continuous location and discrete location models.

Continuous Location Models

Continuous location models choose sites treating latitudes and longitudes as continuous variables. Quantitative analysis of a location problem appeared in 1909 (Weber, 1909). Several factors and approaches have been considered since then. These include the economy of scale factors in the optimal distribution area for a plant/depot (Karnani, 1983). An efficient method for locating warehouses in a cartesian coordinate space has been presented (Murtagh, 1982). A comprehensive account of such models and methods for solving them are discussed by Eilon (Eilon, 1971). There are two limitations to this approach. It is difficult to provide optimus solutions to a continuous problem of moderate size. Also, in practice, potential locations are limited and transportation is not always possible along the Euclidean distances as assumed by continuous models. However, simpler versions can be used to derive insights, and assist in selecting candidate sites.

Discrete Location Models

These models are appropriate when the site locations are selected from a set of potential sites. These models vary in complexity depending on problem features such as multi-commodity, two stage distribution etc. which they include.

A general fixed charge problem has been formulated (Hirsh and Dantzig, 1968). Other formulations include fixed cost transportation problem (Balinski, 1961) and the plant location problem by (Balinski (1964); Efroymson and Ray (1966).

Formulation and Solution Techniques for the Capacitated Warehouse Location Problem

A classical capacitated warehouse location model reads as follows:

Minimize $\sum \sum C_{ij} X_{ij} + \sum F_i Y_i$

Subject to

$X_{ij} = 1$ for all j

$D_{ij} \; X_{ij} \leq S_i Y_i$ for all i

$X_{ij} \geq 0$ for all i, j

$Y_i = 0$ or 1 for all i

where:

Y_i = 1 if warehouse at location i is open , 0 otherwise.
S_i = the capacity of the warehouse at location i.
F_i = the fixed cost of keeping the depot i open
D_j = the demand by customer j.
X_{ij} = the fraction of the jth customer's demand satisfied by warehouse i.
C_{ij} = is the cost of shipping from warehouse i to customer j.

Geoffrion and Graves (1974) discuss a comprehensive formulation, including most of the features listed above, using quadraply subscriped variables. Erlenkotter and Van Roy (1980) discuss a multi-period case. In Spielberg (1979) and Guignard and Spielberg (1979), side constraints (configuration constraints and logical constraints) have been included in the model.

Solution Techniques

Solution techniques of the capacitated warehouse location allocation problem can be classified as: (1) Branch and Bound Method, (2) Bender's Decomposition, (3) Lagrangian Relaxation, (4) Cross Decomposition, (5) Computer Simulation Approach, (6) Heuristic Methods, and (7) Casewise Linear Programming.

Branch and Bound Method

In the Branch and Bound method applied to integer programming problems, the integer requirements are relaxed and a series of L.P. problems solved, the optimum solution being identified when the upper and the lower bounds coincide. This approach can incorporate additional features desired in a depot location problem and has maximum flexibility. Mixed integer programs are N-P complete, that is, computation time and storage requirements increase exponentially, without bounds, as the problem size increases. Many commercial software packages use this method, with special data handling and other utilities to increase efficiency. UMPIRE for Univac 1108, MPSX-MIP for IBM 360 and 370 series, POLYGAMI for ɔC 6000 are examples.

There are variations of branch and bound techniques (Kelly and Khumawala, 1982; Kennington and Unger, 1973, Sa 1969; Pinkus, 1973).

Bender' Decomposition

In this approach the optimum is reached by iterating between two problems: (1) The Master's problem, a pure integer programming problem, and (2) Bender's subproblem, a linear programming problem. The linear programming problem is obtained by fixing the values for the integer variables and the solution provides an upper bound (in a minimization problem). The dual variables of the L.P. generate constraints for the Master's problem in which the values for the integer variables and a lower bound are determined. When the lower bound and the upper bound differ by a predesignated critical value, the algorithm stops.

Lagrangian Relaxation

This approach (Geoffrion and Graves, 1978) uses duality and the Lagrangian multipliers technique. The original problem is relaxed with respect to some constraints to give a relaxed problem. The relaxed constraints, along with the Lagrangian multipliers are incorporated into the objective function, thus penalizing for the relaxation. We iterate between the relaxed problem or the subproblem and the Master problem. Lagrangian multipliers are determined in Master problem and are input for the next iteration of the subproblem. The optimal solution to the relaxed problem gives a lower bound on the optimal solution to the original problem.

For the same problem, different relaxations are possible, depending on the constraints we relax. Generally, a relaxation yielding a tight lower bound on the optimal will be costly to compute, whereas an easily optimized relaxation is likely to result in relatively poor bounds.

In the capacitated warehouse location model by relaxing the demand constraints, the remaining constraints do not couple variables with different i subscripts. Thus, the relaxed problem decomposes into 'm' independent subproblems, one for each 'i'. Each of these can be identified as a 'knapsack problem' which can be readily solved (Geoffrion and McBride, 1978).

Cross Decomposition

A recent paper (Van Roy, 1983) suggested "Cross Decomposition". This combines Bender's Decomposition and Lagrangian Relaxation. In this technique we iterate between a Bender's subproblem and a Lagrangian subproblem and converge to the optimal solution.

Heuristic Methods

Heuristic procedures have also been recommended by many authors, (Armour and Buffa, 1965). Heuristic methods yeild good solutions at reasonable cost and can be used for providing good initial solutions in other optimizing methods.

Casewise Linear Programming

Casewise linear programming can be used in simple problems where a set of design alternatives are evaluated by fixing integer variables and optimizing with respect to the continuous variables. Simulation models have been used to some extent to solve distribution problems (Rardin and Unger, 1976). Computational experience with realistic problems have been reported in several papers (Sa, 1969; Spielberg, 1969; Geoffrion and Graves, 1974; Akinc and Khumawala,). Surveys of algorithms and techniques and detailed bibliography are reported (Salkin, 1975; Geoffrion, 1975).

Comparison of Available Software

The following gives an overview of the available software. To solve the general mixed integer linear programming, the best available commercial systems are UMPIRE for the UNIVAC 1108 MPSX-MIP for the IBM 360 and 370 series, and OPHELIE MIXED for the CDC 600 series. All use the branch and bound strategy, permiting warehouse economics of scale to be taken into account. Matrix generator/report writer packages have been used for distribution planning problems. MULTICOM, was build around OPHELIE MIXED for CDC computers by S.I.A. Another software package known as the DS/SP Generalized Distribution model is primarily intended for use with IBM's MPSX system. POLIGAMI is a software package for the CDC 660 marketed by S.I.A Ltd. for optimization of networks with fixed changes. Facility location problems are perhaps the primary intended area of application. POLIGAMI can accommodate problems with a single product, two stages of distribution, plant capacities and upper limits on warehouse size, and fixed charges and simple economics of scale for warehouses. CAPFLO Is a revised version of a software package developed earlier in England known as DEPLOC and is available through Haverly Systems, Inc. CAPFLO is basically designed to accommodate problems with a single product, two stages of distribution, plant capacities and upper limits on warehouse size.

Elaborate computer simulation packages are available for studying the design and operation of physical distribution systems. DSS (Distribution System Simulator) available from IBM, LREPS (Long Range Environmental Planning Simulator) available through Systems Research Inc. are two examples of the several packages available.

CONCLUSIONS

The models, limitations and computer software for studying distribution planning systems has been reviewed in this paper. The solution techniques were also classified and considered. In reviewing the techniques the excessive computer time required to arrive at a final solution was noted. Further, it was noted that none of the available software packages recognized all of the problem features noted.

The heuristic procedures, which are computationally less intensive, could be developed to provide starting solutions for the more exact methods. The solutions obtained using the heuristics could also provide insights to the final solution

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